1.0 INTRODUCTION

The satellite system of Saturn has been the subject of much research in celestial mechanics since the foundation of this branch of astronomy in its classical form by Laplace in the late 18^{th} century. It has been described as a "solar system in miniature" by virtue of the range of the types of behaviour which characterise the satellite orbits. Within a single system we have

- A very dominant satellite (Titan) which is unaffected by periodic perturbations of a planetary type by its neighbours, and whose motion features only secular perturbations and small periodic solar perturbations.

- Two pairs of satellites (Mimas - Tethys and Enceladus - Dione) whose mean motions are very nearly in the ratio 2:1, causing (among other things) significant librations in the mean longitudes of the satellites concerned.

- A satellite of rather low mass (Hyperion) whose motion is entirely characterised by a close 3:4 resonance with the dominant satellite in the system. The theory of the motion of Hyperion is a problem of such great complexity that Newcomb placed it second only to the lunar theory.

- A satellite (Iapetus) whose theory is dominated by large periodic solar perturbations due to the great distance at which it orbits Saturn. Moreover, the position of the orbit plane of this satellite is governed by long-period perturbations of roughly equal size acting in two widely-separated planes. This means that the secular theory of the node and inclination of Iapetus is of particular interest.

This diversity of behaviour might at first appear daunting, but each satellite (even Titan) only affects its closest neighbours. Most of the satellites are very small and their perturbing effect is only noticeable when it is amplified by a near-resonance. Thus Tethys perturbs Mimas but not Enceladus or Dione, despite the fact that the latter two are its closest neighbours. We may treat the inner satellites (Mimas, Enceladus, Tethys and Dione) as a self-contained system, and likewise the outer satellites, Rhea, Titan, Hyperion and Iapetus. It is the outer satellite system, and in particular the subset consisting of Titan, Hyperion and Iapetus, that is the subject of this thesis. We choose not to include Rhea as an object for direct study, though we shall always be mindful of its perturbations upon the other three satellites.

We begin in chapter 2 with a revision of Sinclair's (1974) theory of the motion of Iapetus in the light of later critical work by Rapaport (1978) and Sinclair and Taylor (1985). Both of these papers note that Sinclair's theory requires improvement and Rapaport investigates a near-resonance with Titan which affects the mean longitude of Iapetus.

During the course of our revision, we find that the principal omissions from Sinclair's theory arise from solar perturbations in the node and inclination. These perturbations have periods of up to 29 years and affect the observed position of the satellite as seen from the Earth by as much as $0''.14$.

We also find that Rapaport overestimates the significance of the Titan quasi-resonance perturbations by a factor of 3, and we present an improved theory of the motion of Iapetus which includes the additional solar and Titan perturbations. The theory is compared with Sinclair and Taylor's (1985) integration of the motion of the outer satellites to obtain a quantitative estimate of the precision of the theory.

Chapter 3 contains a study of the secular motion of the orbit plane of a satellite acted upon by several perturbing forces in different fixed planes. We find that the concept of the Laplacian plane may easily be extended to any number of perturbing forces up to fourth order in the inclinations. We use auxiliary parameters $p = \sin i \sin \Omega$ and $q = \sin i \cos \Omega$ to represent the position of the orbit plane of the perturbed satellite in an arbitrary fixed reference frame and we show that the pole of the orbit plane describes an ellipse about a point which is the pole of the Laplacian plane of the orbit.

This method is applied to the particular case of Iapetus, which is subject to significant perturbations by Titan and the Sun, and smaller perturbations due to the oblateness of Saturn. The orbit of Iapetus is shown to maintain an almost constant inclination of 7° to its Laplacian plane, upon which it precesses with a period of 3000 years. We fit this model to observed values of the node and inclination of the orbit of Iapetus and we determine the mass of Titan as a result of the fitting process.

Chapters 4 and 5 are concerned with the problem of modelling the motion of Titan, Hyperion and Iapetus by numerical integration. This approach has been used by Sinclair and Taylor with some success. They fitted an integration to photographic (astrometric) observations over the period 1967 to 1982 and determined values for the initial position and velocity components of each of the satellites plus the J_2 form-factor and the mass of Saturn and the mass of Titan.

In this thesis we attempt to fit a similar integration to visual (micrometric) observations made during the period 1874 to 1947. Chapter 4 gives an account of the preparation of the raw data for comparison with any dynamical model. This preparation includes the reduction of the various timescales to Universal Time and Ephemeris Time, the calculation of the topocentric position vector of Saturn at the instant of each observation, and an analysis of the effects of stellar aberration and atmospheric refraction upon position angle and separation measures. In addition, we develop the partial derivatives of position angle and separation observations with respect to the Saturnicentric rectangular coordinates of the satellite(s) involved.

Chapter 5 contains a description of the numerical integration method and the procedure employed in fitting the integration to the observations.

It also contains an account of the results of a number of trial iterations in which we attempt to determine the parameters of the satellite system. In particular, values are obtained for the J_2 form-factor of Saturn and the mass of Titan.